Model and formulation in grinding mechanism having advanced secondary rotational axis

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Abstract
“Grinding Mechanism having Advanced Secondary Rotational Axis” is one of the newer plane surface grinding methods that has an uncommon abrasion mechanism. Unlike conventional methods, Grinding Mechanism having Advanced Secondary Rotational Axis has two rotations of a wheel. The first rotation is the same as the conventional grinding methods, which is the circumferential rotation. The other rotation is the newly developed axial rotation, where the wheel rotates around itself perpendicular to its radial axis. In the grinding process, the grinding force, energy, power, and temperature are directly related to the material removal rate. In this article, the chip model in Grinding Mechanism having Advanced Secondary Rotational Axis was addressed and material removal rate was reformulated. The new chip ratio formula was adapted to the grinding force, energy, power, and temperature in the conventional plane surface grinding method. The chip formed in the conventional plane surface grinding method consists of two-dimensional \( xy \) plane. In Grinding Mechanism having Advanced Secondary Rotational Axis, on the other hand, the chips consist of three-dimensional \( xyz \) plane. The reason behind this is the second rotation obtained in Grinding Mechanism having Advanced Secondary Rotational Axis (axial rotational motion). The chip model was obtained from the combination of two rotations in Grinding Mechanism having Advanced Secondary Rotational Axis. As a result, the resulting chip model increased the material removal rate only slightly and this increase was negligible. Accordingly, an increase in grinding force, energy, power, and temperature was observed at negligible rates.

Keywords
Mathematical model in grinding, plane surface grinding, chip model, grinding cut parameters, grinding forces

Introduction
Grinding operation is an important manufacturing method, which allows for obtaining workpieces of desired shape, dimension, and tolerance. This method is especially used in cases where workpieces cannot be manufactured at desired accuracy and surface quality with other methods (turning, milling, etc.).

Chip models and cutting forces have been researched for a very long time. Some of these studies focused on chip model in grinding technique. Other studies focused on cutting parameters in grinding processes. Among these studies, Hecker et al. developed the chip model in Figure 1. The authors studied cutting parameters through this model.

Chips obtained with said chip model were examined and compared to chips obtained with our model and similar images were found, as demonstrated in Figure 2.

Chang and Wang conducted a similar study and modeled the chip cross-section in Figure 3 and performed their experimental study based on this model.

Chip removal with grinding is similar to the milling operation which involves numerous abrasive grains instead of cutter threads. Thickness of cut obtained with an abrasive ranges from zero to maximum cut value, as in milling cutter teeth. The dimension of each chip removed is about a few microns for each abrasive

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However, the chip volume increases considerably since a high number of abrasives remove chips. Another important factor which affects grinding is the wear of abrasives and their breaking from the wheel. Studies show that cutting forces emerge during chip removal and their effects should also be analyzed and measured accurately. Although many studies have been conducted to this end, problems occurring in ground material during the grinding process have not been resolved completely.

Plane surface grinding is a grinding method where the wheel rotates around its own axis in radial direction. This rotating wheel gives different shapes to the workpiece as a result of the combination of linear and circular movements.

Considering the results obtained, the chip form was based on that in usual milling and a chip form similar to that in milling was observed. Various researchers have conducted studies on both wheel structure and grinding methods depending on these factors.

Savaş and Özay investigated effects of cutting parameters on Ra in the new method which they developed, tangential turning-milling, and determined optimum cutting parameters. The authors achieved a very high surface-finishing quality in tangential turn-milling machining of rotationally symmetrical samples. In the case of tangential turn-milling, recorded surface roughness values (Ra) (0.4 μm at 224 r/min spindle speed, at 0.1 mm depth of cutting, and at 530 r/min cutter speed) were lower than that achieved in the case of turning operations.

Other fields of research include the examination of modeling of cutting process in grinding and identification, and modeling of the chip removal process. To this end, researchers examine the wheel structure, number, and distribution of grains and their effects on cutting. Creating a reliable model for the grinding process is a key issue for the estimated performance of the operation. A method based on optical profilometry, representing grinding wheel surface characteristic,
developed. Calculation of the wheel topography information is used in the estimation of surface roughness and grinding force in a computer simulation. Chip removal process simulation of cutting edges of grains accepted as a group is simulated depending on the specific movement between the grinding wheel and the workpiece.

Some researchers simulated the cutting action of each grain engaging with the workpiece and defined a method to investigate the grinding process. Grinding forces are analyzed by simulating the force on each grain which passes a section of the workpiece and the simulated workpiece surface showed features similar in nature to the experimental results. The sinking condition caused by the grain on the workpiece surface was also examined in this study. The grain modeling was performed with a spherical structure and cutting forces obtained from sinking and scraping shapes were also addressed. Calculations were compared with experimental workpieces and similar results were found.

"Grinding Mechanism having Advanced Secondary Rotational Axis" (GMASRA) is one of the newer plane surface grinding methods offering a newly developed mechanism. For this reason, this method has been examined from different perspectives. In previous study, the conventional plane surface grinding technique and the GMASRA grinding technique were compared in terms of the surface quality of the ground parts. In the conventional plane surface grinding technique, different Ra values were measured in settings where the wheel was parallel, perpendicular, and angled with respect to the feed direction. In the GMASRA technique, the Ra values were the same throughout the workpieces, in each of their region, regardless of the feed direction. Using the same cutting parameters and wheel types, the Ra values in grinding the AISI 1015 material were lower in the GMASRA technique than in the conventional plane surface grinding technique. In another study of GMASRA, the same comparison was made by changing the parameters. As a result, better Ra values were obtained again from the GMASRA technique. Equal Ra values were obtained in each region of the workpieces and in each direction of roughness measurement. It was observed that the wheel surface was equally worn in both methods. In another study related to GMASRA, the motor power required for the first rotational motion used in the GMASRA technique and the second rotational motion added to the system was modeled by using the MATLAB program. The required cutting speed for the ideal grinding operation in the model was 25 to 30 m/s, the diameter of the grinding wheel was chosen in the range of 75 to 325 mm, and the required number of revolutions and motor power were calculated for both rotational movements. A similar study calculated the grinding force of the second rotational motion in the system by using a mathematical modeling method. The force equations in the experimental studies were obtained from mathematical equations of the conventional grinding method. The effects of the determined parameters on the grinding force were calculated separately, and graphs were obtained regarding the change by using the MATLAB program. Finally, in the recent study conducted with GMASRA, the GMASRA and conventional grinding methods were compared, using the Taguchi orthogonal test design, by selecting the most effective parameters. The surface roughness (Ra) values were measured after the conventional grinding and the GMASRA methods. The most quality surfaces were obtained in horizontal direction, direction of proceeding of grinding wheel, in the conventional grinding method. However, higher surface roughness values were recorded both in vertical and diagonal directions. Therefore, the authors investigated that there were differences in surface roughness in the classical grinding method and GMASRA.

It is seen in the literature that grinding processes are generally carried out using conventional methods. The chip obtained in the GMASRA grinding technique needs to be examined. It is necessary to compare and formulate the grinding parameters, related to the grinding forces, power, energy, and so on, of the GMASRA grinding method with the conventional surface grinding method. In this study, a grinding model, which is an alternative to conventional grinding machines, was developed, and a prototype of it was mounted on a CNC vertical machining center. Chip models were examined in GMASRA method. A comparison was made with the chip form obtained in the conventional grinding method and grinding force, energy, power, and temperature formulas in the new grinding method have been formulated again. For this formulation, a comparison was made between the conventional surface grinding method and the GMASRA grinding method depending on the material removal rate (MRR). As a result, it was seen that there is very little difference between the GMASRA grinding method and the conventional surface grinding method in terms of the MRR.

Chip model analysis

Grinding is a very complex and versatile method, using to improve the dimensional accuracy of samples. In grinding processes, shapes and numbers of the grain are not regular. Therefore, the subject was investigated both experimentally and analytically by the researchers. The data related to the experimental studies were obtained from the plane surface grinding tests and the analyses of the results. Analytical studies can be categorized into three different areas: these are the statistical approach, energy approach, and physical-based approach. In the present study, specific issues, such as abrasive particle size and its ratio, were not discussed. The main aim is to determine the effective rate
of GMASRA, on the grinding force and other grinding parameters, based on any mathematical model. For this purpose, the path followed by the grain was chosen as the main parameter. In the GMASRA method, the differences in the length of the path followed by the grain were calculated. Therefore, MRR was calculated and the formulas were adapted to the GMASRA method in the selected model.

In addition, in the study conducted by JS Colton, the author examined cross-section of chips obtained from the grinding process and performed mathematical modeling for grinding forces, temperature, and energy in relation to chip removal rate and formulated the grinding process. As a result, chip cross-section obtained with plane surface grinding process was examined and a model was developed based on the two-dimensional chip model, as seen in Figure 4.

In GMASRA, the modeling in the two-dimensional chip model developed by Colton was preserved, as demonstrated in Figure 5(a). In addition, a new model was developed in this study with chip volume obtained based on chip cross-section, and path and calculations were made for grinding forces, energy, and temperature.

There are two rotations in GMASRA as explained in detail in the first article: circumferential and axial rotations. Wheel and grain motions in the classic method can be seen in Figure 5(a), and grinding wheel and grain motions in GMASRA can be seen in Figure 6(b).

When wheel motions in Figure 5(a) are examined, two-dimensional shape of chip obtained with the classical grinding method is shown in Figure 6(a). As seen in Figure 6(a), the chip path forms two-dimensional \(x-z\) plane. Here, \(z\) direction is the function of cutting depth and the \(x\) direction is the function of the linear feed rate of workpiece. Here, linear feed rate is calculated in relation to wheel's circumferential speed. Thus, both cutting parameters are calculated, and the chip model is developed and a two-dimensional function is obtained.\(^1\,^2\)

In this study, on the other hand, a three-dimensional parabolic chip path forms due to axial rotation (second rotation–spindle rotation) of grinding wheel, as shown in Figure 6(b).

As seen in Figure 6(b), the chip path forms in \(x-y-z\) plane as a result of GMASRA, unlike the classical grinding method. Here, similar to the classical grinding method, \(z\) direction is the function of depth of cut and the \(x\) direction is the function of the linear feed rate of workpiece. Unlike the classic grinding method, a separate chip path occurs as a result of the axial rotation in \(y\) direction. The \(y\) direction occurs as a result of the axial rotation of the wheel. As composite function, a three-dimensional function occurs \(x\) direction depending on feed rate of workpiece, \(z\) direction depending on depth of cut, and \(y\) direction depending on axial rotation.

In the classical grinding method, calculations of grinding force, energy, and temperature depend on parabolic chip path forming on \(xz\) plane as in Figure 6(a). MRR obtained depending on parabolic chip path is one of the most important parameters here and all these calculations are derived from MRR. Similarly, all grinding force, energy, and temperature calculations are produced depending on MRR in GMASRA as well. MRR, on the other hand, depends on the three-dimensional chip path seen in Figure 6(b). For this reason, a modeling is necessary to calculate the MRR according to chip cross-section obtained with three-dimensional chip path.

**Materials and methods**

In order to derive a mathematical modeling, it is necessary to examine the shapes of the chips in GMASRA method. Thus, the similarity of the chip, in the classical
grinding method, the plane surface of the chip can be examined. For this purpose, in the present experimental study, AISI 1015 steel was preferred as samples. The samples were prepared in dimensions of 100 mm × 10 mm × 80 mm (height × thickness × width). Therefore, the GMASRA mechanism is fastened on a CNC table shown in Figure 7. The dimensional properties of the CNC machine, employed in experimental processes, are shown in Table 1.

Five different kinds of grinding wheels were used to abrade the chips. For each wheel, three different kinds of depth of cut and axial rotation speeds, but two different steps over distance, were selected. Grinding parameters and wheel properties are shown in Table 2.

In order to examine the chip shape obtained with GMASRA, chips coming out during the grinding process were collected. To this end, the grinding process was performed in a dry environment and chips were taken from the paper laid on the flange. Collected chips were put between slides and examined under microscope as seen in Figure 8.

The microscope used here was a NIKON Eclipse TS 100F model microscope, which can be seen in Figure 9. Photos were taken with 1/40 magnification under microscope. It was seen that chip shapes imaged were similar to the literature, as demonstrated in Figure 10.

**Discussion**

**Calculation of 3D chip length (X) and cutting speed (V)**

In the previous chapter, we noted that the chip shape was three-dimensional parabolic in x, y, and z axes shown in Figure 6(b). First, it is necessary to know the chip size (X) in order to calculate grinding force, power, energy, and temperature for the model to be developed.

Basic elements in classical grinding models are shown in Figure 11.

Figure 11 shows linear length of arc length (Xr), theoretically calculated from X, arc length forming on the workpiece and grinding wheel interface of the workpiece in a two-dimensional model.

The symbols in Figure 11 are as follows:

- X: Workpiece and grinding wheel contact arc length (mm)
- Xr: Workpiece and grinding wheel contact arc length and beam length (mm)
- D: Wheel diameter (mm)
The length of $X_r$ arc can be calculated, depending on the beam angle ($\theta$) as derived in equations (1) and (2)

\[
\frac{D}{2} \cdot \cos \theta = \frac{D}{2} - d \Rightarrow \cos \theta = \frac{D - d}{D} \Rightarrow \\
\theta = \cos^{-1} \left(1 - \frac{2d}{D}\right)
\]

\[
X_r = \pi \cdot \frac{D}{360} \cdot \theta \Rightarrow \quad X_r = \pi \cdot \frac{\text{acos} \left(1 - \frac{2d}{D}\right)}{360}
\]

In addition, $X_r'$ arc length can be calculated depending on $X_r$, arc length, according to the geometrical profile, demonstrated in Figure 12, as identified in equation (3)

\[
(X_r')^2 = \left(\frac{D}{2} \cdot \sin \theta\right)^2 + d^2 \Rightarrow (X_r')^2 = \frac{D^2}{4} \cdot \sin^2 \theta + d^2 \\
+ d^2 \Rightarrow (X_r')^2 = \frac{D^2}{4} \cdot (1 - \cos^2 \theta) + d^2 \\
(X_r')^2 = \frac{D^2}{4} \cdot \left(1 - \left(1 - \frac{2d}{D}\right)^2\right) + d^2 \Rightarrow (X_r')^2 = D \cdot \frac{d}{D} - d^2 + d^2 \Rightarrow X_r' = \sqrt{D \cdot d}
\]

In order to obtain the GMASRA model, it is necessary to know the length difference between $X_r$ and $X_r'$. Results related to $X_r$ and $X_r'$ obtained from equation (3) are shown in Table 3.

As can be seen in Table 3, there is not a considerable difference between $(X_r)$ and $(X_r')$ for equal depths of cut. This is a tolerable value. For this reason, calculations will hereafter be performed using the $X_r'$ value by the help of equation (3) for convenience.

The following method was followed to calculate the chip removal rate used in the calculation of force, power, energy, and temperature. The motion created by the axial rotation in the system and associated elements are required to be determined for the chip cross-section and chip removal rate in GMASRA. The relevant three-dimensional representation can be found in Figure 12.

As shown in Figure 12, there are two different angular speed and linear speed in the system. Similar to the classical grinding method, there are two different speeds: radial linear speed of the wheel ($V_r$) and the linear speed of the wheel ($\omega_r$). These are the first group of speeds. In addition, perpendicular angular speed ($\omega_e$) and linear speed ($V_e$) occur as a result of the axial rotation in GMASRA. Elements specified in Figure 12 are as follows:

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**Table 2. Wheel types used and grinding parameters.**

<table>
<thead>
<tr>
<th>Wheel type</th>
<th>Abrasive material</th>
<th>Grain size</th>
<th>Hardness</th>
<th>Texture</th>
<th>Binder</th>
<th>Properties of wheel</th>
<th>Grinding parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>NK</td>
<td>36</td>
<td>P</td>
<td>S</td>
<td>V</td>
<td>$d = 20$ mm</td>
<td>Value 1  Value 2 Value 3</td>
</tr>
<tr>
<td>Type 2</td>
<td>NK</td>
<td>46</td>
<td>O</td>
<td>S</td>
<td>V</td>
<td>$D = 75$ mm</td>
<td>80           160       240</td>
</tr>
<tr>
<td>Type 3</td>
<td>NK</td>
<td>60</td>
<td>N</td>
<td>S</td>
<td>V</td>
<td>$B = 16$ mm</td>
<td>4            8         0.01      0.02       0.03</td>
</tr>
<tr>
<td>Type 4</td>
<td>EKR</td>
<td>46</td>
<td>K</td>
<td>6</td>
<td>V</td>
<td>Stepover (mm)</td>
<td>300</td>
</tr>
<tr>
<td>Type 5</td>
<td>EKR</td>
<td>60</td>
<td>K</td>
<td>6</td>
<td>V</td>
<td>Speed of the wheel (r/min)</td>
<td>7015</td>
</tr>
</tbody>
</table>

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**Figure 7.** GMASRA mechanism mounted on CNC vertical machining center.

- $d$: Depth of cut (mm)
- $\theta$: Beam angle identifies $X_r$ and $X_r'$ dimensions (degree).

The length of $X_r$ arc can be calculated, depending on the beam angle ($\theta$) as derived in equations (1) and (2)

\[
\frac{D}{2} \cdot \cos \theta = \frac{D}{2} - d \Rightarrow \cos \theta = \frac{D - d}{D} \Rightarrow \\
\theta = \cos^{-1} \left(1 - \frac{2d}{D}\right)
\]
Angular speed \( v \) and cutting speed of the grinder \( (V) \) can be calculated as demonstrated in equation (4)

\[
\omega = \frac{2 \pi n}{60} \quad \text{rad/s} \quad \text{and Linear speed}: \quad V = \omega r \text{ m/s}
\]

(4)

Figure 13 shows the radial linear speed \( (V_r) \) and angular speed \( (\omega_r) \) obtained with the circumferential rotation of the wheel.

The radial angular speed of the wheel, shown in Figure 14, needs to be calculated with the help of equations (4) and (6). Furthermore, the linear speed of the wheel \( (V_r) \) can be calculated as demonstrated in equation (5)

\[
V_r = \omega_r \frac{D}{2}
\]

(5)
When the axial rotation of the wheel is examined, a conical motion occurs depending on axial spindle rotation. This is shown in Figure 14.

According to the geometrical profile, seen in Figure 14, spindle axis linear speed \( V_e \) can be calculated as demonstrated in equation (6)

\[
V_e = \omega e \sqrt{\frac{D^2}{2} - \left(\frac{D}{2} - d\right)^2}
\]

where \( r \) is the axial effective rotation radius (mm).

In GMASRA, due to both radial (circumferential) motion and axial rotation of the wheel, it is necessary to

1. find the length of cut by calculating the parabolic path in the chip wheel contact interface of the obtained parabolic chip cross-section and calculate the chip removal rate using length of cut, and
2. calculate energy, power, and temperature using the calculated chip removal rate.

It was previously shown in Table 1 that the difference between the length of \( X_r \) and \( X_r' \) obtained from equations (3) and (4) was not considerable. Therefore, assuming the three-dimensional parabolic path of the
The following can be written from the triangle in Figure 16: if the $X_r$ formula from equation (4) is combined with equation (13), it can be written as follows. With the help of the $X_r$ and $X_e$ triangle, as shown in Figure 15, linear three-dimensional length of cutting can be calculated, as seen in equations (4) and (14).

Once the necessary operations are performed, the formula of the parabolic length of cut $(X)$ which is formed by the composite of the two rotations can be expressed.

**Calculation of MRR**

A model will be created based on Colton’s\textsuperscript{13} chip model to calculate MRR. MRR is the main determinant in the calculation of power, energy, and temperature. It is necessary to find MRR to obtain all these values.

Assuming the chip cross-section in Figure 4 occurs as a triangular prism and expressing the ratio ($r$) of chip width ($w$) to thickness ($t$) as $r$, demonstrated in equation (9)

$$r = \frac{w}{t} \approx 10 \text{ to } 20$$

In GMASRA, this ratio will be used for the chip cross-section obtained in Figure 6(a) as well. Therefore, chip volume (MRR) is calculated, as identified in equation (10)
\[ MRR = \frac{1}{2} w \cdot \frac{1}{2} t \cdot l = \frac{1}{4} w \cdot t \cdot l \] (10)

The calculation of chip volume as a function of time is as follows:

\[ l: \text{Chip of cut (mm)} \quad \beta: \text{Angle between } \mathbf{V_e} \text{ and } \mathbf{V_r} \text{ vectors (degree)} \]

\[ w: \text{Chip width (mm)} \quad c: \text{Number of cutting edges} \]

\[ b: \text{Width of cut (mm)} \quad V: \text{Cutting speed (m/s)} \]

\[ d: \text{Depth of cut (mm)} \quad x: \text{Length of cut (mm)} \]

\[ v: \text{Feed rate (m/min)} \quad r: \text{Ratio of chip width to chip thickness} \]

Assuming \( V \) is the circumferential speed of the wheel, the rotating speed of the grinder can derive as in equation (11)

\[ n = V \cdot b \cdot c \] (11)

With taking both equations (10) and (11) into consideration, the MRR is obtained and also the circular speed of the grinder is derived as in equation (12)

\[ MRR = v \cdot d \cdot b = n \cdot Volume_{chip} \Rightarrow v \cdot d \cdot b = V \cdot b \cdot c \cdot \frac{1}{4} w \cdot t \cdot l \Rightarrow w = r \cdot t \] (12)

where \( l \) symbol is the length of cut. In GMASRA, on the other hand, \( X \) was used for length of cut as seen in equation (8). If we replace \( l \) with \( X \) in equation (12), the thickness of cutting can be demonstrated as seen in equation (13)

\[ v \cdot d \cdot b = V \cdot b \cdot c \cdot \frac{1}{4} w \cdot t \cdot l \Rightarrow v \cdot d \cdot b = V \cdot c \cdot b \cdot \frac{1}{4} r \cdot t \cdot t \]

\[ t = \sqrt{\frac{D \cdot d + \frac{4 \omega e^2 \cdot d^2 \cdot (D - d)}{\omega r^2 \cdot D}}{V \cdot c \cdot r \cdot \sqrt{D \cdot d + \frac{4 \omega e^2 \cdot d^2 \cdot (D - d)}{\omega r^2 \cdot D}}}} \] (13)

To find the thickness of cut in the grinding process is of great importance to calculate grinding energy, grinding force, grinding force used for each abrasive grain, and grinding temperature. All these parameters increase or decrease in direct proportion to thickness of cut.

**Energy formation in grinding process and grinding force**

Energy formed in the grinding process is the sum of chip energy, friction energy, and slip energy. This expression is given in equation (14)

\[ u = u_{chip} + u_{friction} + u_{slip(mm)} \] (14)

The total grinding force can be calculated, as in equation (15)

\[ Power = u \cdot MRR \Rightarrow F_{grinding} = V = u \cdot v \cdot d \cdot b \] (15)

Experimental studies show that increase in chip energy is proportional to decrease in friction energy, as written in equation (16)

\[ u = \frac{1}{l} \text{ or } u = K_1 \cdot \frac{1}{l} \] (16)

With solving equations (15) and (16) together, the total grinding force \( F_{grinding} \) can be demonstrated as seen in equation (17)

\[ F_{grinding} = K_1 \cdot \frac{1}{l} \cdot \frac{v \cdot d \cdot b}{V} \] (17)

In addition, with replacing the cutting thickness \( t \) into equation (17), the total grinding force \( F_{grinding} \) value can be calculated as derived in equation (18)

\[ F_{grinding} = K_1 \cdot \sqrt{\frac{V \cdot c \cdot r \cdot \sqrt{D \cdot d + \frac{4 \omega e^2 \cdot d^2 \cdot (D - d)}{\omega r^2 \cdot D}}}{4 \cdot v \cdot d}} \Rightarrow F_{grinding} = K_1 \cdot b \cdot \frac{d \cdot v \cdot c \cdot r \cdot \sqrt{D \cdot d + \frac{4 \omega e^2 \cdot d^2 \cdot (D - d)}{\omega r^2 \cdot D}}}{4 \cdot V} \] (18)

**Calculation of power for a single grain**

The power, each grain subjected to grinding system, can be calculated using equation (19)

\[ F_{grain} = u \cdot Area \Rightarrow F_{grain} = u \cdot \frac{1}{2} w \cdot t \] (19)

\[ \Rightarrow w = r \cdot t \Rightarrow u = K_1 \cdot \frac{1}{l} \]

\[ F_{grain} = K_1 \cdot \frac{1}{l} \cdot \frac{1}{2} r \cdot t \cdot t \] (20)

If the value of \( t \), as demonstrated in equation (13), is written in equation (20), the grain force \( F_{grain} \) can be derived as identified in equation (21).

It was stated that the chip cross-section obtained depending on the wheel–surface interface was three-dimensional parabolic and thickness of cut was calculated as seen in equation (20). For this reason, if \( t \) thickness of cutting demonstrated, as in equation (13), replaced in equation (21), the force \( F_{grain} \) each grain subjected to can derived, as identified in equation (21)

\[ F_{grain} = K_1 \cdot \frac{v \cdot r}{V \cdot c} \cdot \sqrt{\frac{1}{D \cdot d + \frac{4 \omega e^2 \cdot d^2 \cdot (D - d)}{\omega r^2 \cdot D}}} \] (21)

This formula is used in GMASRA to calculate the grinding force of each abrasive grain.
**Temperature in grinding process**

The calculation of temperature in the grinding technique is performed, depending on grinding area. This temperature is calculated, as written in equation (22), depending on grinding area:

\[
\Delta T = K_2 \frac{ua.b.l.d}{b.t} = K_2 K_1 \frac{1}{d} (22)
\]

As seen in equation (22), the temperature depends on thickness of cut \( (t) \) and depth of cutting \( (d) \). Furthermore, the general temperature, in grinding operations, can be calculated using equation (23):

\[
\Delta T = K_2 K_1 \sqrt[4]{\frac{V_{c.r}}{4v_{y}}} \sqrt{D.d + \frac{4\omega e^2 d^2(D - d)}{\omega^2 d}}
\]

\[
\Rightarrow \Delta T = K_2 K_1 \sqrt[4]{\frac{V_{c.r} d}{4v_{y}}} \sqrt{D.d + \frac{4\omega e^2 d^2(D - d)}{\omega^2 d}} (23)
\]

There are any differences between the path, followed by the grain in the GMASRA method and the conventional grinding method, as seen in equation (8). In parallel with the increase in the length of the path followed by the grain, MRR increases as well. The change in MRR leads to an increase in ground forces, the process temperature, and the required power. However, there is no excessive increase in these parameters (the required power and energy, the process temperature) due to the fact that the length of the path followed by the grain does not increase too much in GMASRA.

On the other hand, MRR has an effect on Ra. A large number of studies have been carried out on the relationship between the increase in MRR and Ra. The MRR increases due to the increase in values such as feed rate, depth of cut, cutting speed, and spindle speed. Depending on the MRR, there is an increase in Ra. Varma found that the increase in the depth of cutting and the feed rate, cause to reduced Ra, but with increasing the cutting speed Ra values decreased. They found that the prediction model developed using regression analysis, neural networks, and adaptive-neuro fuzzy inference system (ANFIS) had the same trend between MRR and Ra. The same results were obtained in the cylindrical grinding of AISI 1040 steel.

The grinding of the demand for sapphire \( (\alpha-Al_2O_3) \) material by the electrolytic in-process dressing (ELID) and the results obtained from this experiment in terms of Ra and MRR were examined. There is no direct relationship between MRR and Ra. In the present experiment, the effect of various values on the Ra and MRR was investigated.

An optimization modeling developed, using ANN and genetic algorithm. The greatest effective parameter on Ra was the spindle speed of workpiece (37%), followed by the depth of cut (37%) and the feed rate (24%), respectively, on cylindrical grinding. The effect of grinding parameters on the Ra at conditions \( V_w = 10,000 \text{ mm/min} \) and \( d = 0.01 \text{ mm} \) in soft grinding and at conditions \( V_w = 20,000 \text{ mm/min} \) and \( d = 0.02 \text{ mm} \) in aggressive grinding was investigated.

In this study, various roughness values were generated with selecting different dressing conditions at the beginning of grinding (once the wheel is freshly dressed). This was also experimentally investigated in the tests. Furthermore, it was observed that when aggressive dressing conditions were applied, the roughness always tended to worsen until reaching a steady state value, that is, around 2.5 to 3 mm for all cases.

Evolution of roughness values was shown in different trends, contrary to the presented, depending on the dressing conditions, with applying soft grinding conditions. Soft dressing conditions would worsen the roughness as wheel wear progressed, while more aggressive dressing conditions would tend to improve roughness as wheel wears out. Esmaeilzare determined the increase in surface roughness with the increase in the feed rate, depth of cut, and a decrease in the surface roughness with the increase of the cutting speed.

Surface roughness (Ra) was increased in cylindrical grinding by increasing the depth of cut, the feed rate, and the increase in the workpiece speed. Ra also increased with the increase of MRR.

Aslan and Budak developed a thermo-mechanical model to predict forces in grinding with circumferentially grooved and regular (non-grooved) wheels. In this model, the path followed by the grain was taken as the basis. With the increase in the path followed by the grain, there was an increase in these forces and the increase in the number of helical grooves on the wheel was observed in Ra.

Similar to all the above studies, as the depth of cut increases in the GMASRA method, the path of the particle increased, as written in Table 3. All studies show that Ra tends to increase linearly with the increase of MRR. Previous studies on the GMASRA method showed a decrease in Ra in the grinding process with the GMASRA method. In the GMASRA method, the Ra values were the same throughout the samples, in each of their region, regardless of the feed direction. This result is different to the previous researches. In our previous study on GMASRA, the most effective parameter on Ra was studied. For this purpose, different wheel types, depths of cut, stepover, and axial rotational speed parameters were analyzed, and finally axial rotational speed was detected to be the most effective parameter. As a result, a decrease in Ra was observed with axial rotation added in GMASRA.

**Conclusion**

In this research, the chip formula obtained with the new rotation is the same compared with the chip model developed by other researchers, yet only the length of cut is observed to be higher. A three-dimensional parabolic chip model resembling a comma was obtained in
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References